

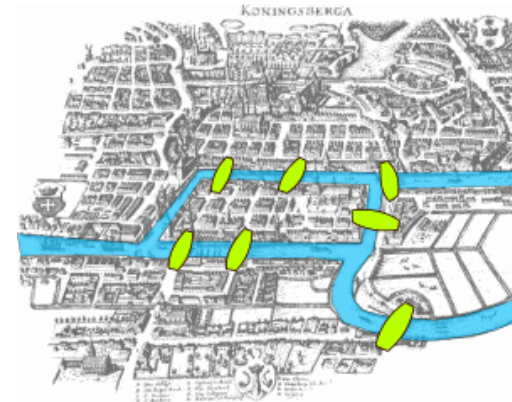
7. Graphs

Discrete Mathematics 2010/11

Yipee!

Logic Sets Relations Functions Counting Induction Graphs

Seven Bridges of Königsberg



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Seven Bridges of Königsberg

During the 18th century (300 years ago) there was a city called Königsberg in Russia (now called Kaliningrad).

The city was 4 islands connected by 7 bridges.

The people liked to take long walks on Sundays.

They asked: "Is it possible to take a walk that crosses every bridge only once?"

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Seven Bridges of Königsberg

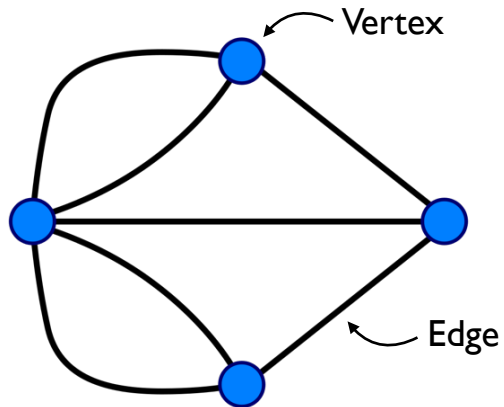
The Königsberg bridges problem was solved by Euler.

He represented the problem using vertices and edges.

His work became known as Graph Theory!

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Seven Bridges of Königsberg



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Set of vertices

+

=

Graph

Set of edges

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Graphs

Definition:

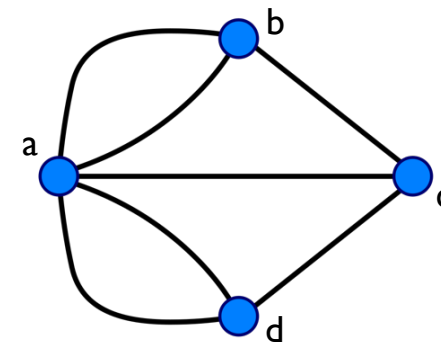
A *graph* $G = (V, E)$ consists of a set of *vertices* V and a set of *edges* E ,

where each edge is a pair connecting two vertices

$E : V \Leftrightarrow V$ (E is a subset of $V \times V$).

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Graphs



$G = (V, E)$

$V = \{a, b, c, d\}$

$E = \{(a,b), (b,a), (b,c), (a,c), (a,d), (d,a), (c,d)\}$

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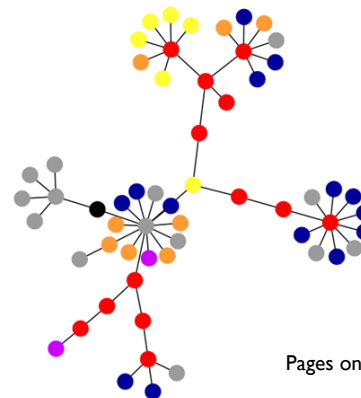
Examples of graphs



London Tube Map

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Examples of graphs

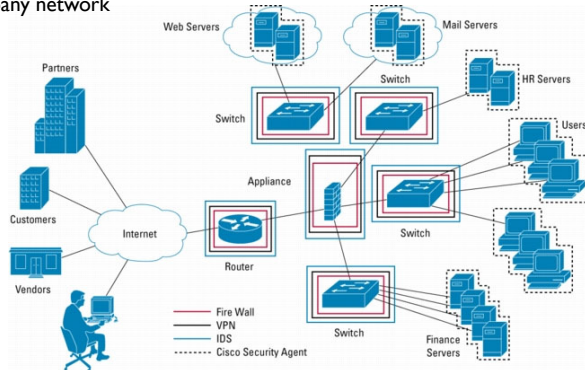


Pages on google.com

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Examples of graphs

Company network



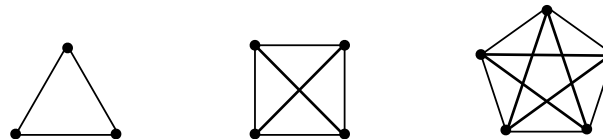
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Properties of graphs

Definition:

A graph is *complete* if every vertex has an edge to every other vertex.

Examples:



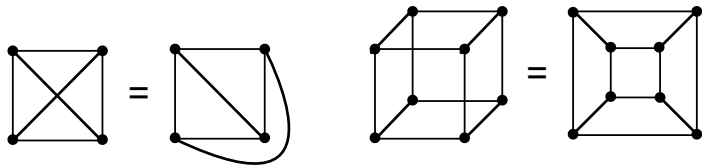
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Properties of graphs

Definition:

A graph is called *planar* if it can be drawn without any lines crossing/intersecting.

Examples:



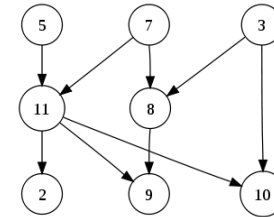
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Directed graphs

So far, we have only seen *undirected graphs*.

A *directed graph* is a graph where the edges have a direction.

Example:

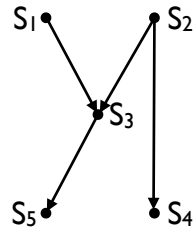


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Directed graphs

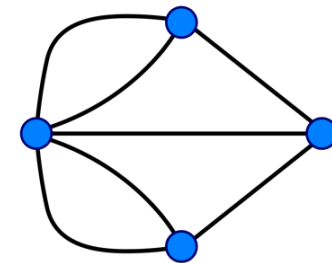
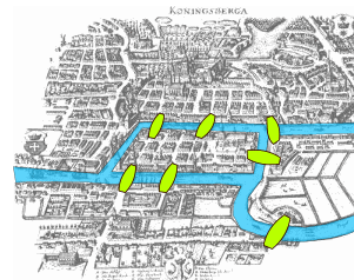
Used to create *precedence graphs* of programs:

- S₁ a = 0;
- S₂ b = 1;
- S₃ c = a + b;
- S₄ a = b;
- S₅ b = c;



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Back to bridges



Can you make a walk along every bridge/edge only once?

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Back to bridges

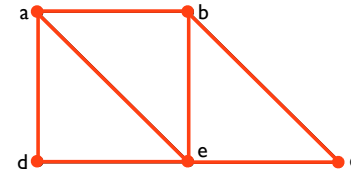
Answer: No, there is no possible walk.

Why?

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Euler paths and circuits

An *Euler path* is a walk in a graph that visits every **edge** exactly once.

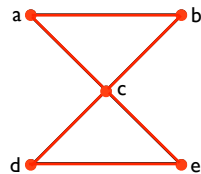


Euler path: a, d, e, c, b, e, a, b

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Euler paths and circuits

An *Euler circuit* is an Euler path that starts and ends at the same vertex.

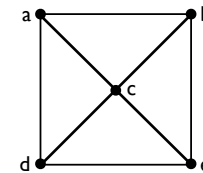


Euler circuit: a, b, c, d, e, c, a

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Euler paths and circuits

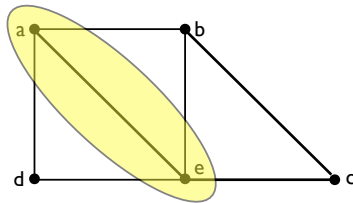
The following graph has no Euler path:



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Euler paths and circuits

A graph is an Euler path if only two vertices have an odd number of edges.

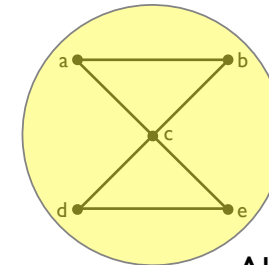


Odd number of edges

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Euler paths and circuits

A graph is an Euler circuit if every vertex has an even number of edges.

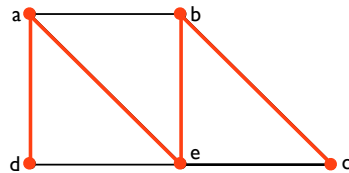


All even edges

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Hamilton paths and circuits

A *Hamilton path* is a walk in a graph that visits every **vertex** exactly once.

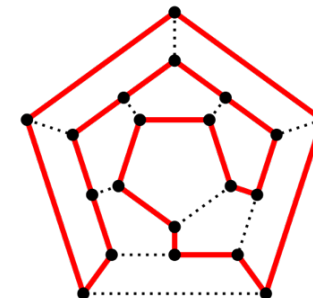


Hamilton path: d, a, e, b, c

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Hamilton paths and circuits

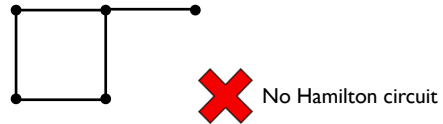
A *Hamilton circuit* is a Hamilton path that starts and ends at the same vertex.



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Hamilton paths and circuits

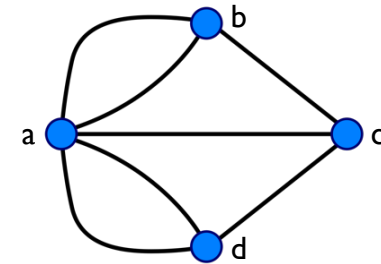
No simple method for determining whether a graph has a Hamilton path or circuit!



But if a vertex only has one edge then the graph cannot have a Hamilton circuit.

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Back to bridges



The Konigsberg bridges puzzle has no Euler path or circuit.

But, it has a Hamilton circuit: a, b, c, d

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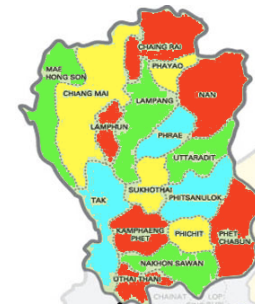
Graph colouring



Can you colour the provinces so that two neighbouring provinces do not have the same colour?

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Graph colouring

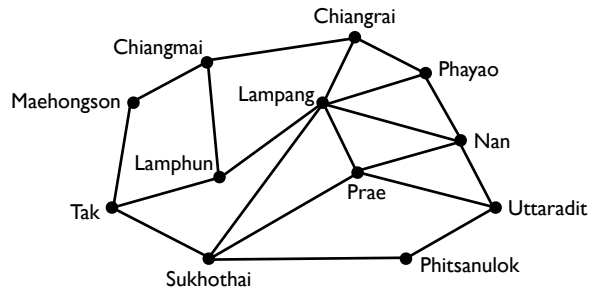


Yes
With 4 or more colours

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Graph colouring

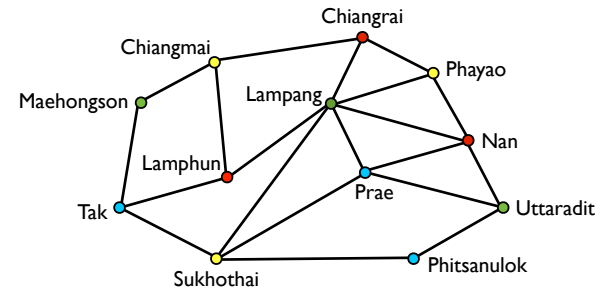
Any map can be drawn as an planar graph:



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Graph colouring

And then coloured...



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The four colour theorem

Theorem:

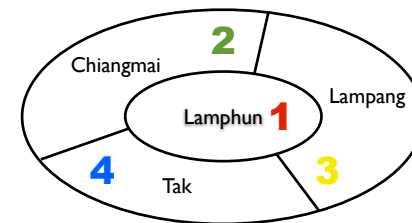
Any planar graph can be coloured using at most 4 colours.

or

Any map can be coloured using at most 4 colours.

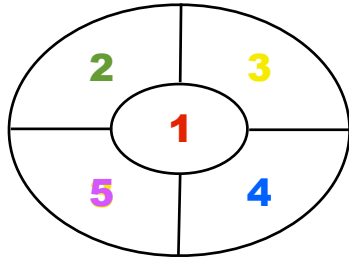
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The four colour theorem



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The four colour theorem



Only ever need 4 colours with planar graphs

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Vocabulary

Graph Vertex / vertices Edge / edges

Complete graph

Planar graph

Directed graph

Euler circuits & Euler paths

Hamilton circuits & Hamilton paths

Graph colouring

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The End!