Workshop 1: Logic

1. Which of the following are statements?

   a. Papaya salad is spicy.

   b. How are you today?

   c. Computer scientists love discrete mathematics.

   d. If I was an animal then I would be a monkey.

   e. Please mind the gap between the train and the platform.

2. Write down the truth tables for implication and equivalence, then:

   a. Use truth tables to prove that $(A \Leftrightarrow B) \Leftrightarrow C$ is the same as $A \Leftrightarrow (B \Leftrightarrow C)$.

   b. Is the same true for $(A \Rightarrow B) \Rightarrow C$ and $A \Rightarrow (B \Rightarrow C)$? Use truth tables to prove it.

   c. Consider a statement S with n variables. How many rows would the truth table for S have?

3. Use the laws of logic (and not truth tables) to simplify the following statements:

   a. $\neg(\neg A \wedge \neg B) \wedge (\neg A \vee B)$

   b. $\neg(A \Rightarrow A) \Rightarrow (A \Rightarrow (B \Rightarrow \neg A))$

   c. $(\neg A \vee B) \Rightarrow ((B \Rightarrow A) \vee (A \Rightarrow B))$

4. Which of the following are either a tautology or a contradiction? Explain why.

   a. $(A \wedge \neg A) \Rightarrow B$

   b. $\neg(A \vee B) \wedge A$

   c. $A \vee B$

   d. $(\neg A \wedge C) \Rightarrow (B \vee C)$

5. In computer hardware, it is common to find a logic gate called NAND. For A NAND B, the statement is false when both A and B are true, else it is false.

   a. How can you write NAND using the boolean operators $\neg$ and $\wedge$?

   b. Use laws of logic to show an equivalent statement using only $\neg$ and $\Rightarrow$.

   c. NAND ($\uparrow$) is a special boolean operator because you can rewrite any boolean statement using only the NAND operator. Show (using truth tables or laws of logic) that $\neg$, $\wedge$, $\vee$ and $\Rightarrow$ can be expressed using only $\uparrow$.