Workshop 2: Logic

1. Which of the following statements cannot be expressed in propositional logic?

   a. The car is black.
   b. Not all cars are red.
   c. All birds can fly.
   d. The teacher likes to ask questions.
   e. There exists some students who fall asleep in class.

2. Let S(x) be the predicate “x is good looking” and let T(x) be the predicate “x comes from Sukhothai” and the domain is “students at NU”. Write the following statements in English.

   a. S(Bart Simpson)
   b. $\neg$S(Homer Simpson) $\wedge$ $\neg$T(Homer Simpson)
   c. $\forall x : S(x)$
   d. $\exists x : \neg T(x)$
   e. $\forall x : T(x) \Rightarrow S(x)$

3. Let C(x) be the predicate “x has a cat” and let D(x) be the predicate “x has a dog” and the domain is “students at NU”. Write the following sentences in predicate logic.

   a. No students at NU have a cat.
   b. There exists at least one student at NU who has a dog and a cat.
   c. All students at NU have either a dog or a cat.
   d. If all students at NU have a dog then there does not exist any student with a cat.

4. Consider the predicate $Q(x, y, z) \Leftrightarrow x * y = z + 1$ where $x, y, z \in N$. Determine whether the following statements are true or false:

   a. Q(3,4,13);
   b. $(Q(3,2,1) \vee Q(3,2,1)) \Leftrightarrow Q(2,2,2)$;
   c. $Q(1,2,2) \Rightarrow Q(2,4,3)$;
   d. $\forall x \in N: \forall y \in N: \exists z \in N: Q(x,y,z)$;
   e. $\forall x \in N: \forall y \in N: \forall z \in N: Q(x,y,z)$;

5. Write each statement using predicate logic, and then determine its truth value.

   a. There exists a natural number that is greater than 5.
   b. Every natural number is the sum of 3 natural numbers.
   c. Every natural number is the sum of 3 distinct natural numbers.
   d. There exists a natural number that is not the sum of 2 distinct natural numbers.

6. Show that $\forall x : (P(x) \Rightarrow Q(x))$ is not equivalent to $\forall x : P(x) \Rightarrow \forall x : Q(x)$.