

## Workshop 4: Sets

1. Describe, in English, the meaning of:

- a. A set
- b. The empty set
- c. A power set

2. Using Venn diagrams, show that:

- a.  $A \cup B = (A \cap B) \cup (A \setminus B) \cup (B \setminus A)$
- b.  $(A \cup B) \setminus (A \cap B) = (A \setminus B) \cup (B \setminus A)$

3. Write the following sets using the set builder notation.

- a.  $\{0, 3, 6, 9, 12\}$
- b.  $\{-3, -2, -1, 1, 2, 3\}$
- c. The set of fruits that grow in Thailand.
- d. The set of real numbers between one and ten.

4. What is the cardinality of the following sets?

- a.  $\{x \in \mathbb{N} \mid x^2 < 100\}$
- b.  $P(\{a, b, \{a, b\}\})$
- c.  $P(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

5. Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{0, 3, 6\}$ . Find:

- a.  $A \cup B$
- b.  $A \cap B$
- c.  $A / B$
- d.  $P(B / A)$
- e.  $|P(A) \times B|$

6. Using laws of logic, prove that:

- a.  $(A \cup B) \cap (B \cup C) = B \cup (A \cap C)$
- b.  $\overline{A \cap B} = \overline{A} \cup \overline{B}$

7. Using laws of sets, show that:

a.  $(A \cap B) \cup (A \cap \bar{B}) = A$

b.  $(B / A) \cup (C / A) = (B \cup C) / A$

c.  $(A / C) \cap (C / B) = \emptyset$

8. The *symmetric difference* of A and B (written as  $A \oplus B$ ) is the set containing elements in A and B but not in both A and B. In set builder notation:

$$A \oplus B = \{ x \mid (x \in A \wedge x \notin B) \vee (x \notin A \wedge x \in B) \}$$

a. Draw a Venn diagram of  $A \oplus B$ .

b. What is the symmetric difference of  $\{1, 3, 5\}$  and  $\{1, 2, 3\}$ ?

c. Prove or disprove that  $A \oplus B = (A / B) \cup (B / A)$ .