

Workshop 6: Functions (and more relations)

1. Which of the following relations are reflexive, symmetric, antisymmetric or transitive? Explain why.

a. $R_a : \{1,2,3,4\} \leftrightarrow \{1,2,3,4\}$

$$R_a = \{ (1,1), (2,2), (3,3) \}$$

b. $R_b : \{1,2,3,4\} \leftrightarrow \{1,2,3,4\}$

$$R_b = \{ (1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3) \}$$

c. $R_c : \mathbb{N} \leftrightarrow \mathbb{N}$

$$R_c = \{ (x,y) \in \mathbb{N} \times \mathbb{N} \mid y = x + 1 \}$$

2. Consider the relation $R_{\text{letters}} : \text{Words} \leftrightarrow \text{Words}$ (again!) where two words are related if they have the same number of letters.

This week, let $\text{Words} = \{ \text{Hello, Tata, chickens, love, discrete, maths, goodbye} \}$.

a. Draw a diagram of this relation.

b. How many equivalence classes are there in this relation?

3. In the set Words , change “Tata” for your own name.

a. Draw a new diagram for this relation.

b. Write down every equivalence class.

4. Which of the relations in question 1 are functions?

5. Which of the following functions are surjective, injective, or bijective?

a. $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(x) = x^2$$

b. $f : \{1,2,3\} \rightarrow \{1,2,3\}$

$$f = \{ (1,2), (2,3), (3,1) \}$$

c. $f : \{a,b,c,d\} \rightarrow \{ \text{milk, toast, cornflakes} \}$

$$f = \{ (a,\text{milk}), (b,\text{milk}), (c,\text{toast}), (d,\text{toast}) \}$$

d. $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(x) = \text{floor}(x/2) \quad (\text{the floor function takes any real number and returns the integer part})$$

e. $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$

$$f(x,y) = x + y$$