

Workshop 9: Induction

- Let $P(n)$ be the statement $1^2 + 2^2 + 3^2 + \dots + n^2 = n(n+1)(2n+1)/6$ where n is a natural number.
 - What is the value of $P(4)$?
 - Show that the base case $P(0)$ is true.
 - Write the inductive step.
 - Show that the inductive step is true.
- Using proof by induction, show that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ is true for all positive integers.
- Prove that $1*2 + 2*3 + 3*4 + \dots + n(n+1) = n(n+1)(n+2)/3$ is true for all positive integers.
- Using mathematical induction, prove that $2^n < n!$ ($!$ = factorial) for all positive integers greater than 3 ($n > 3$).
- Write a recursive function for $\text{SumSequence}(n) = 1 + 2 + 3 + \dots + n$. For example:
 $\text{SumSequence}(1) = 1$ $\text{SumSequence}(2) = 3$ $\text{SumSequence}(3) = 6$
- Write a recursive function that returns the n th Fibonacci number. For example:
 $\text{Fibonacci}(1) = 1$ $\text{Fibonacci}(2) = 1$ $\text{Fibonacci}(3) = 2$ $\text{Fibonacci}(4) = 3$
- Write an *iterative* function that returns the n th Fibonacci number.

Bonus question (for real computer scientists!)

- The following function, called `Search`, is similar to the recursive function `Contains` (from the lecture).

```
int Search(int[] array, int x, int startposition) {  
    ???  
}
```

Implement the function `Search` such that it returns the position of the number x in the array (or -1 if not found). This is called a linear search. For example:

$\text{Search}(\{2,5,6,3,4\},5,1) = 2$ $\text{Search}(\{2,5,6,3,4\},4,1) = 5$ $\text{Search}(\{2,5,6,3,4\},8,1) = -1$